Universal quantum control of two qubits

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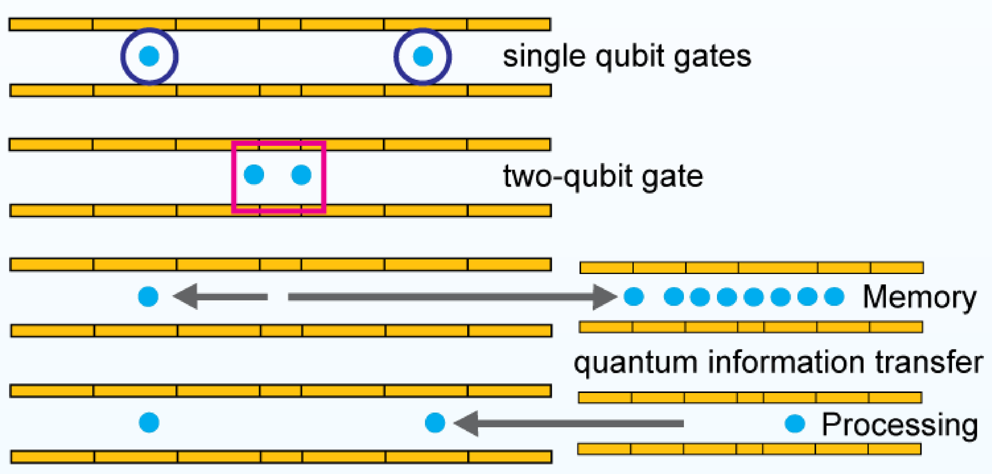
IARPA

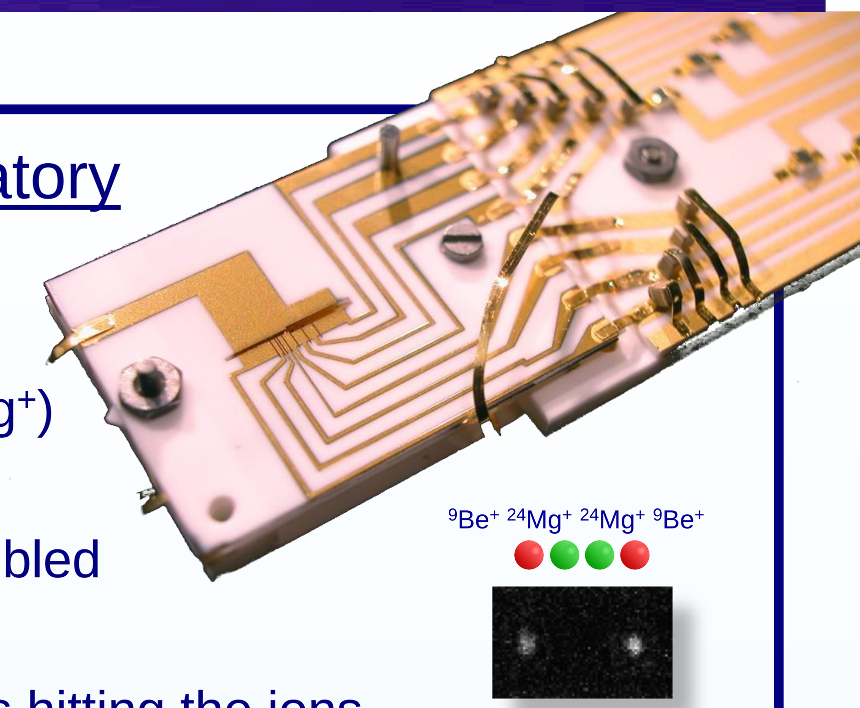
Quantum Computation

- Capable of simulating any physical system[1]
- Capable of accelerating certain calculations[2]
- One and two-qubit gates are universal[3]
- Our scheme, the ion-trap quantum CCD[4]

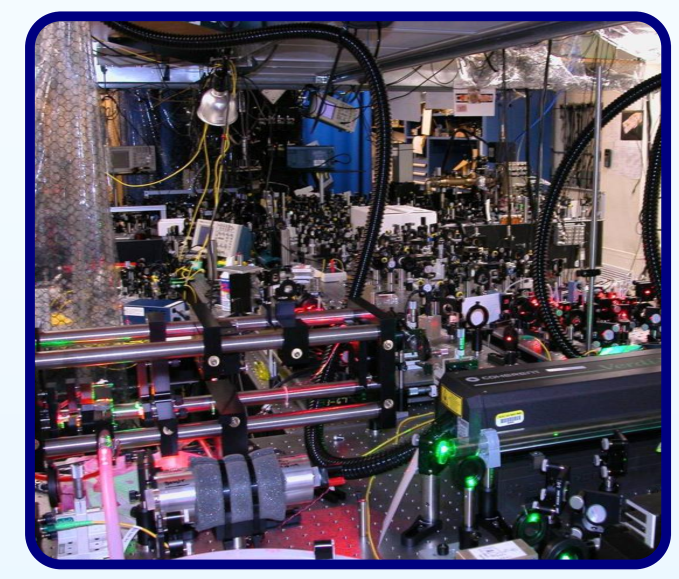


The universal quantum computer is a device that could simulate any physical system and represents the major goal for the field of quantum information science. Such a device requires the ability to perform all possible unitary transformations in the system's Hilbert space. Here we demonstrate universal control of a four-dimensional quantum system. We implement a quantum algorithm that realizes any unitary two-qubit operation up to a physically irrelevant global phase. Using quantum state and process tomography, we characterize the fidelity of our implementation for a large number of randomly chosen unitaries. The methods used here are scalable to higher dimensional Hilbert spaces.

The Laboratory



- Multizone trap
- Two ion species ($^9Be^+$ and $^{24}Mg^+$)
- Five dye lasers frequency-doubled to the UV
- 16+ laser beams hitting the ions
- 30 PID loop filters to stabilize laser frequencies and intensities
- Field-programmable gate array (FPGA) control of laser pulses and rf frequencies/phases
- Laser frequencies fine-tuned using rf from direct digital synthesis (DDS) chips
- Custom GUI and programming language for experiment control

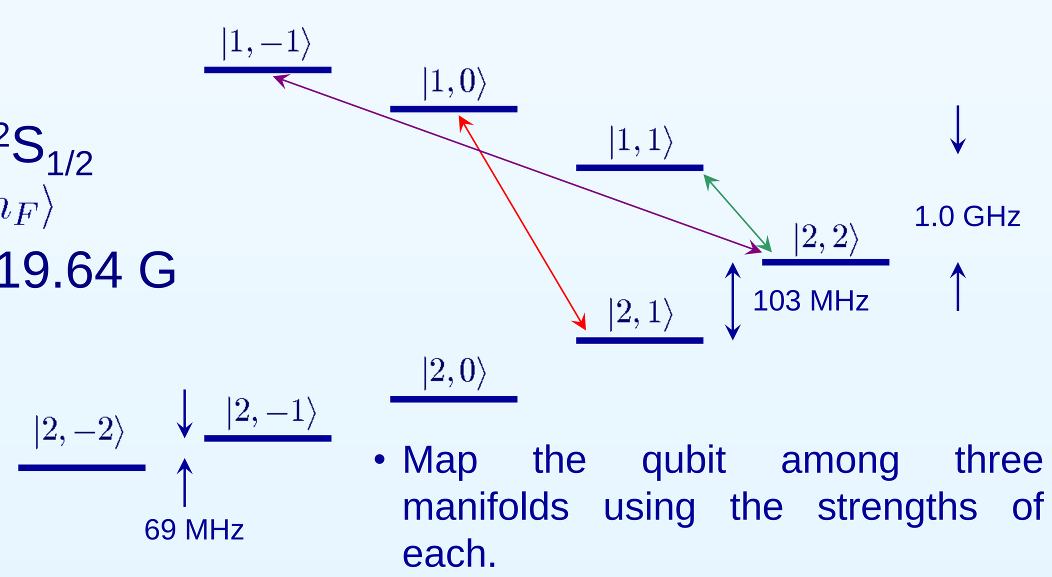


$^9Be^+$ Hybrid Qubit Storage

$^9Be^+$ $^2S_{1/2}$

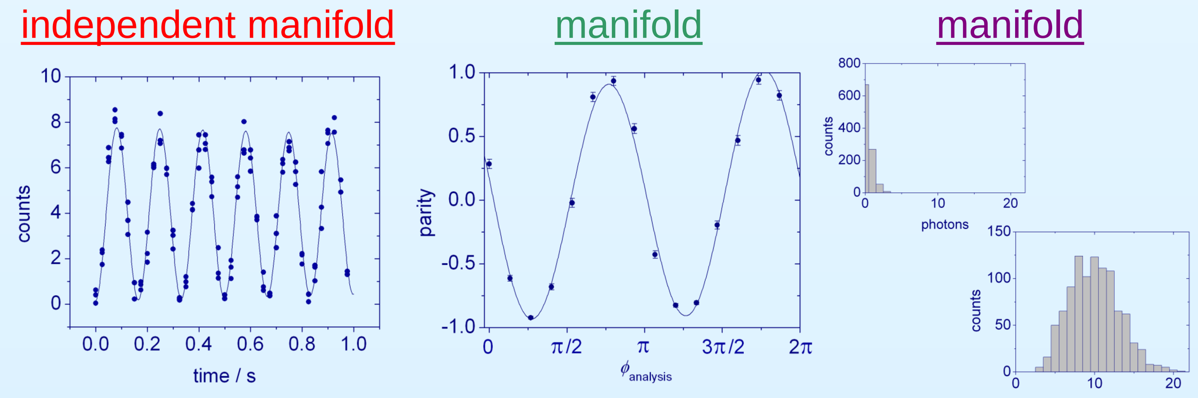
$|F, m_F\rangle$

B = 119.64 G



- Map the qubit among three manifolds using the strengths of each.
- Spectroscopically resolved

Magnetic-field-independent manifold | Two-qubit gate manifold | Measurement manifold



Coherence time[5] ~ 15 s

$$\frac{\partial\omega}{\partial B} = 0$$

Sizeable differential a.c. Stark shift[6]

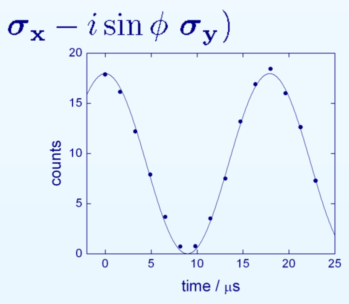
State-dependent resonance fluorescence[4]

Universal Gate Set

Single-qubit π/2 rotations

$$R\left(\tfrac{\pi}{2},\phi\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -ie^{-i\phi} \\ -ie^{i\phi} & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\mathbf{I} - i\cos\phi\,\boldsymbol{\sigma_x} - i\sin\phi\,\boldsymbol{\sigma_y}\right)$$

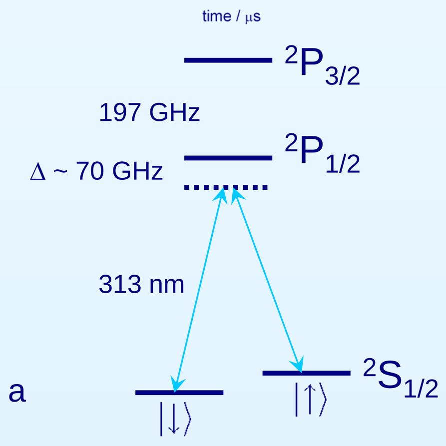
- Coherent stimulated Raman transition[4]
- Adjust the laser phase relative to the qubit
- Individual addressing through separation



Single-qubit *z* rotations

$$R_z(\phi) = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} = \cos\tfrac{\phi}{2}\mathbf{I} - i\sin\tfrac{\phi}{2}\,\boldsymbol{\sigma_z}$$

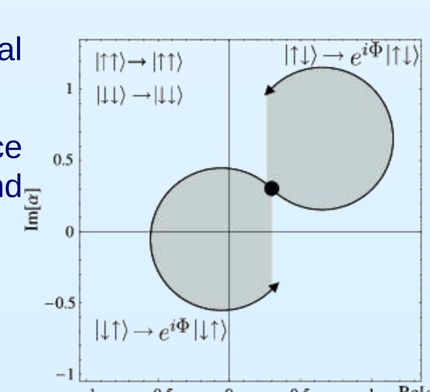
- Step the laser phase relative to the qubit



Two-qubit geometric phase gate[6]

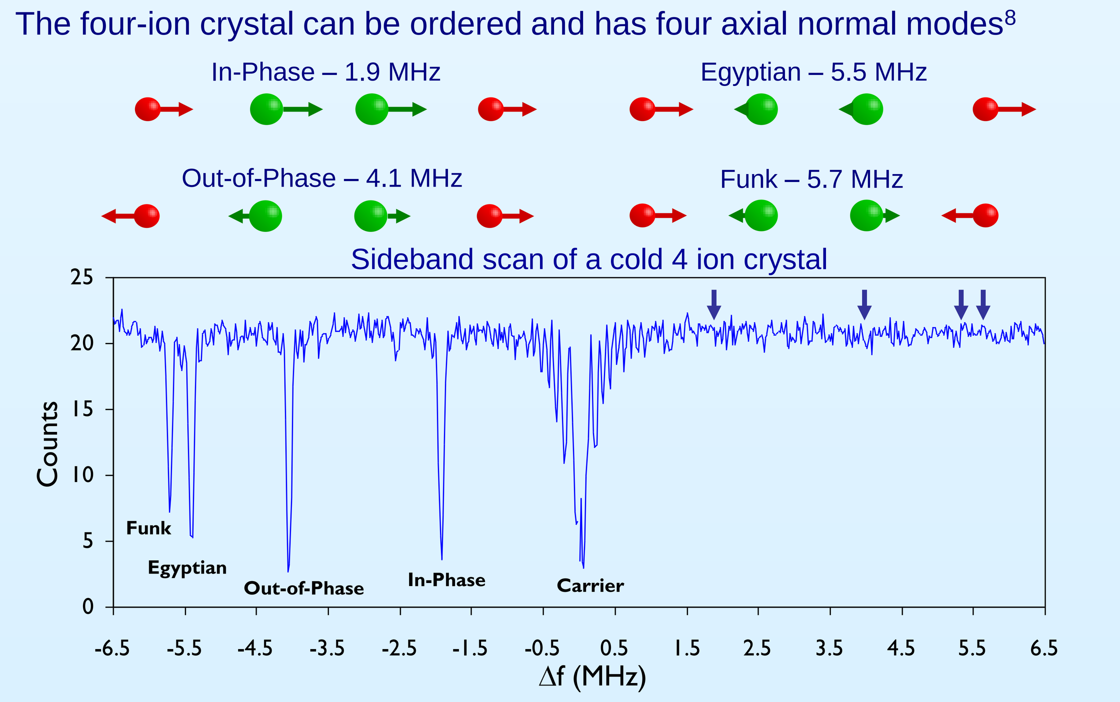
- State-dependent optical dipole force
- Drive around a loop in phase space to add a phase
- State-dependence from differential a.c. Stark shift
- Tune between the highest frequency axial modes to resonantly excite them
- The ion spacing controls the phase of the force (e.g. space by odd-λ/2 to drive only |↓↑⟩ and |↑↓⟩ on a symmetric mode).

$$\hat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



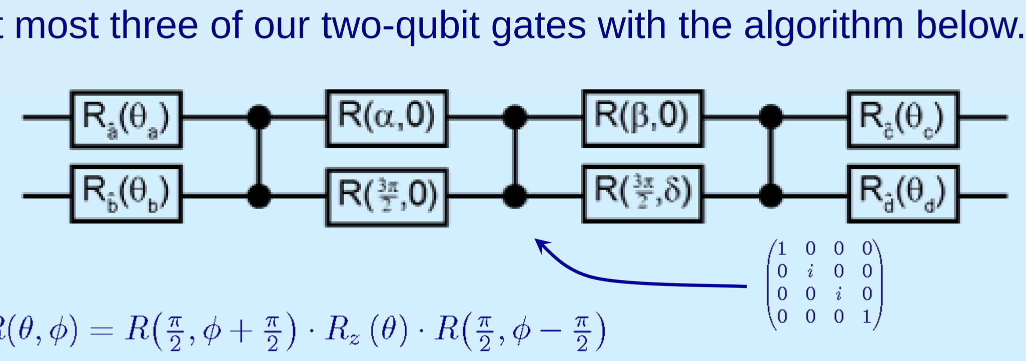
$^{24}Mg^+$ Sympathetic Cooling

- Doppler cool before each single-qubit gate
- Resolved-sideband cool before each two-qubit gate $\bar{n} \sim 0.06$
- Does not affect qubit coherence[7] (280 nm versus 313 nm)
- The four-ion crystal can be ordered and has four axial normal modes[8]



Synthesizing Arbitrary Operations

All two-qubit operations are members of the group SU(4) up to a global phase. All members of this group can be realized with at most three of our two-qubit gates with the algorithm below.



$$R(\theta,\phi) = R\left(\tfrac{\pi}{2},\phi+\tfrac{\pi}{2}\right)\cdot R_z(\theta)\cdot R\left(\tfrac{\pi}{2},\phi-\tfrac{\pi}{2}\right)$$

Steps to universal control

1. Pick a two-qubit operation, e.g., a random matrix in SU(4) with Haar measure
2. Find an operation in its local equivalence class, i.e., identical up to single-qubit operations (3 degrees of freedom)
3. Find the single-qubit operations (12 degrees of freedom)

This can be done analytically using local invariants of two-qubit gates when represented in the "magic" Bell basis[9,10]:

$$\frac{1}{\sqrt{2}}\left(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\right), \frac{i}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right), \frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right), \frac{i}{\sqrt{2}}\left(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle\right)$$

For example, the operations M, $L \in$ SU(4) are equivalent up to single-qubit operations iff $M_B^T M_B$ and $L_B^T L_B$ have the same eigenvalues.

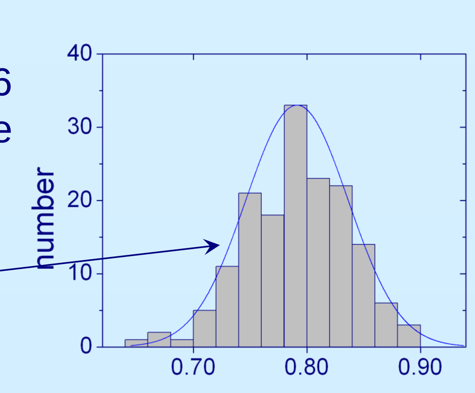
Tomographic Data

State Tomography

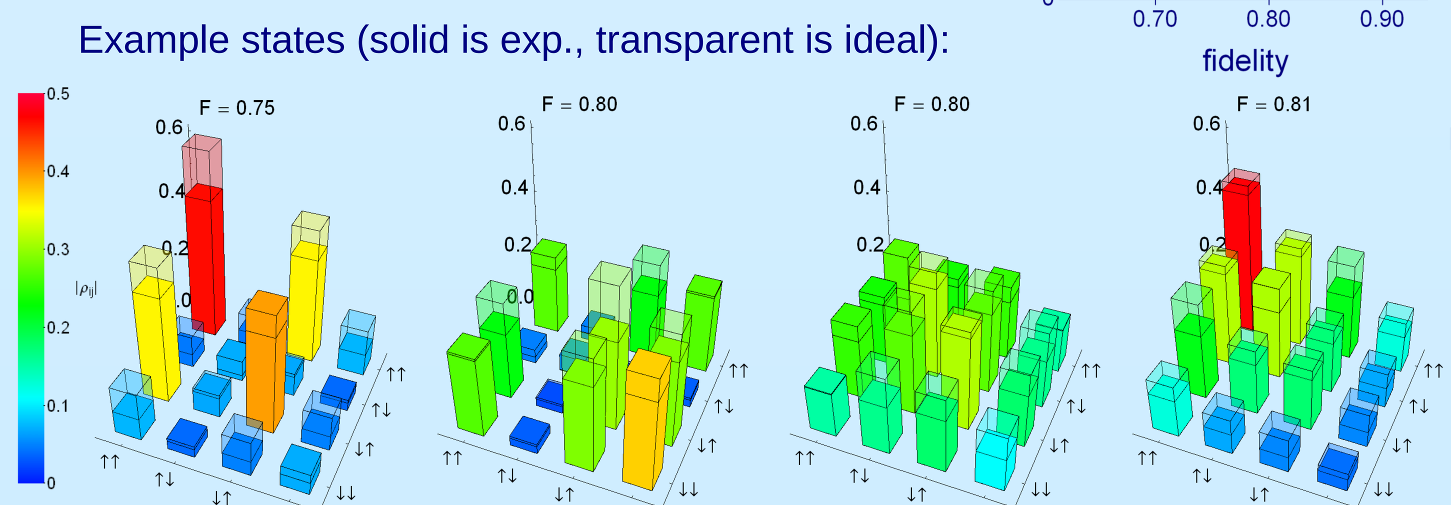
Apply 10 randomly chosen unitaries each to 16 orthogonal input states and conduct state tomography on the outcomes.

Measure a mean state fidelity of

$$\bar{f}_{\Upsilon\in SU(4)} = 79(5)\%$$



Example states (solid is exp., transparent is ideal):



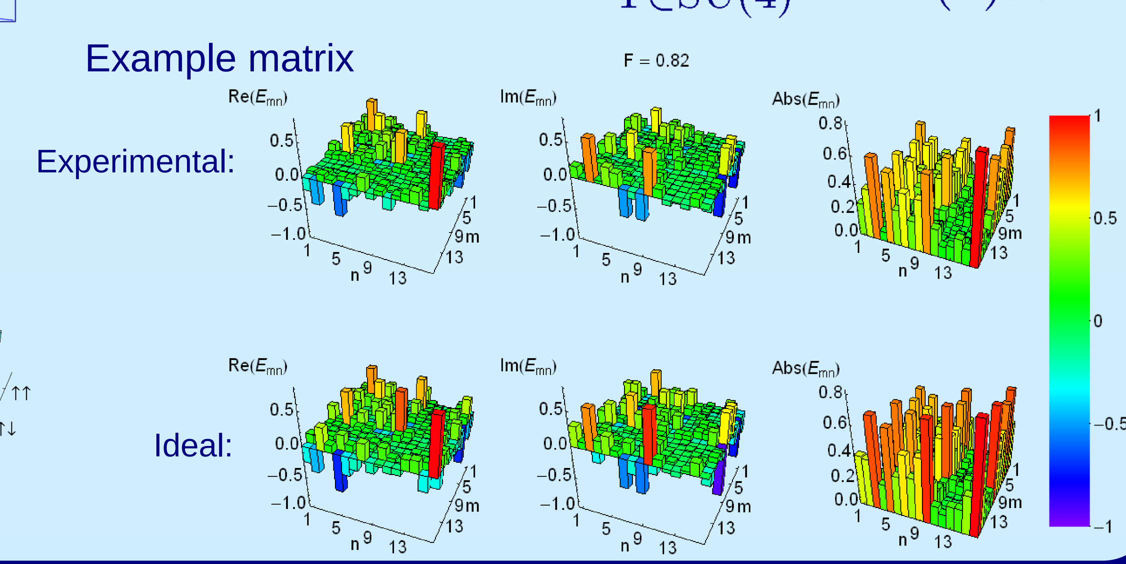
Process Tomography

Obtain the process matrix[11] for 11 of these by analyzing all 16 input states in 9 measurement bases.

Measure a mean process fidelity of

$$\bar{F}_{\Upsilon\in SU(4)} = 79(3)\%$$

Example matrix



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