Universal quantum control of two qubits

NIST Ion Storage Group, Time & Frequency Division, Boulder, CO
David Hanneke, Jonathan Home, John Jost, Jason Amini, Dietrich Leibfried, David Wineland

Quantum Computation
- Capable of simulating any physical system\(^1\)
- Capable of accelerating certain calculations\(^2\)
- One and two-qubit gates are universal\(^3\)
- Our scheme, the ion-trap CCD\(^4\)

The universal quantum computer is a device that could simulate any physical system and represents the major goal for the field of quantum information science. Such a device requires the ability to perform all possible unitary transformations in the system’s Hilbert space. Here we demonstrate universal control of a four-dimensional quantum system. We implement a quantum algorithm that realizes any unitary two-qubit operation up to a physically irrelevant global phase. Using quantum state and process tomography, we characterize the fidelity of our implementation for a large number of randomly chosen unitaries. The methods used here are scalable to higher dimensional Hilbert spaces.

The Laboratory
- Multizone trap
- Two ion species (\(^{9}\text{Be}^+\) and \(^{24}\text{Mg}^+\))
- Five dye lasers frequency-doubled to the UV
- 16+ laser beams hitting the ions
- 30 PID loop filters to stabilize laser frequencies and intensities
- Field-programmable gate array (FPGA) control of laser pulses and rf frequencies/phases
- Laser frequencies fine-tuned using rf from direct digital synthesis (DDS) chips
- Custom GUI and programming language for experiment control

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Universal Gate Set

**Single-qubit \(\pi/2\) rotations**
- \(R_x(\phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi} \\ e^{-i\phi} & 1 \end{pmatrix}\)
- \(R_y(\phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} & -ie^{i\phi/2} \\ -ie^{-i\phi/2} & e^{i\phi/2} \end{pmatrix}\)
- \(R_z(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}\)

**Single-qubit \(z\) rotations**
- \(R_z(\phi) = e^{i\phi/2} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix}\)

**Two-qubit geometric phase gate**
- State-dependent optical dipole force
- Drive around a loop in phase space to add a phase due to the \(\pi/2\) phase operation

**State Tomography**
- Apply 10 randomly chosen unitaries each to 16 orthogonal input states and conduct state tomography on the outcomes.
- Measure a mean state fidelity of \(F_{\text{ESU}(4)} = 79(5)\%\)

**Tomographic Data**
- Example states (solid is exp., transparent is ideal):

**Process Tomography**
- Obtain the process matrix\(^{21}\) for 11 of these by analyzing all 16 input states in 9 measurement bases.
- Measure a mean process fidelity of \(F_{\text{PESU}(4)} = 79(3)\%\)

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Synthesizing Arbitrary Operations

All two-qubit operations are members of the group SU(4) up to a global phase. All members of this group can be realized with at most three of our two-qubit gates with the algorithm below.

\[
R_{12}(\alpha, \beta) = R_{21}(\beta, \alpha) R_{12}(\beta, \alpha) R_{12}(\alpha, \beta)
\]

Steps to universal control:
1. Pick a two-qubit operation, e.g., a random matrix in SU(4) with Haar measure
2. Find an operation in its local equivalence class, i.e., identical up to single-qubit operations (3 degrees of freedom)
3. Find the single-qubit operations (12 degrees of freedom)

This can be done analytically using local invariants of two-qubit gates when represented in the “magic” Bell basis\(^{10}\).

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References