New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

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Abstract. A new measurement resolves the cyclotron and spin levels for a single-electron quantum cyclotron to obtain a dimensionless electron magnetic moment, g, to 7.6 parts in 10^{13} (nearly six times better than in the past) and shifted by 1.7 standard deviations. The new g, with a quantum electrodynamics (QED) calculation, determines the fine structure constant with a 0.7 ppb uncertainty – ten times smaller than for atom-recoil determinations. Remarkably, this 100 mK measurement probes for internal electron structure at 130 GeV.

Keywords: electron magnetic moment, electron g value, fine structure constant, quantum cyclotron
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1. INTRODUCTION

Two of the most fundamental constants of nature are measured much more accurately than ever before – the dimensionless form of the electron magnetic moment [1] and the fine structure constant [2]. The new methods required to achieve such big improvements, developed over 20 years, are summarized here.

2. NEW MEASUREMENT OF THE ELECTRON g

Measurements of the electron magnetic moment (μ) probe for internal electron structure and probe the electron's interaction with the fluctuating vacuum of QED. As an eigenstate of spin **S**, the electron (charge $-e$ and mass m) has $\mu \propto \mathbf{S}$,

$$\mu = -g\frac{e\hbar}{2m}\frac{\mathbf{S}}{\hbar}. \tag{1}$$

The g value is a dimensionless measure of the moment, with the dimensions and approximate size given by the Bohr magneton, $e\hbar/(2m)$. If the electron was a mechanical system with an orbital angular momentum, then g would depend upon the relative distributions of the rotating charge and mass, with $g = 1$ for identical distributions. (Cyclotron motion of a charge in a magnetic field B, at frequency $\nu_c = eB/(2\pi m)$, is one example.) A Dirac point particle has $g = 2$. QED predicts that vacuum fluctuations and polarization slightly increase this value. Electron substructure [3] would make g deviate from the Dirac/QED prediction (as quark-gluon substructure does for a proton).

Measurements of the electron g have a long history [4, 5], with a celebrated measurement [6] providing the accepted value [7] since 1987. Our new g is slightly shifted with a six time smaller uncertainty (Fig. 1a). A one-electron quantum cyclotron [8], cavityinhibited spontaneous emission [9], a self-excited oscillator (SEO) [10], and a cylindrical Penning trap [11] contribute to the extremely high precision. For the first time, the lowest cyclotron and spin levels of a single electron are fully resolved via quantum non-demolition (QND) measurements [8], and a cavity shift of g is directly observed.

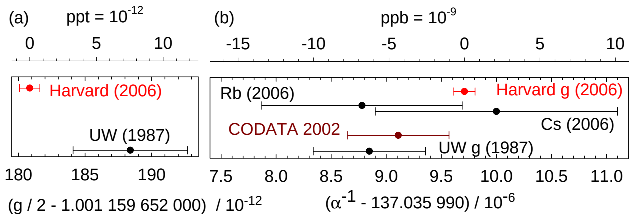

FIGURE 1. Six-fold improved electron g is 1.7 standard deviations from the last measurement (a). New α is ten times less uncertain than values of α deduced from Rb and Cs [12, 13], and the current CODATA value [7] (b). Measured g are converted to α with current QED theory.

What can be learned from a more accurate electron g? The first result beyond g itself is a shifted and much less uncertain value for the fine structure constant, $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ [2] – about ten times more accurate than nearest rival methods. This is the first uncertainty reduction since 1987 in this measure of the strength of the electromagnetic interaction, a crucial ingredient in our system of fundamental constants [7]. Second, the most stringent test of QED theory is provided, limited by ten times larger uncertainties in independent measurements of α. Third, even though muon g values have nearly 1000 times larger uncertainties [14], the muon and electron g values together are a sensitive probe for physics beyond the standard model. Calculations of the muon g depends more sensitively upon heavy particles, as well as upon α, which the electron g provides.

3. BIG IMPROVEMENT REQUIRES NEW METHODS

One electron suspended in a Penning trap is used for the new measurement, like in past. However, methods developed for the new measurement over 20 years (and reported in 6.5 Ph.D. theses) make it possible to realize an electron that resides entirely in the quantum ground state of its cyclotron motion. This “quantum cyclotron” leaves its ground state only when we deliberately send a resonant photon to excite it. The homemade “atom” that we make for the measurement is now essentially quantum mechanical rather than classical.

1. Only the lowest quantum levels of the spin and cyclotron motion are used for the measurement. The cyclotron frequency, as well as the spin frequency, is measured using quantum jump spectroscopy.
2. A Penning trap that is also a microwave cavity is used to control the radiation field. It can suppress spontaneous emission from the cyclotron motion by more than a factor of 100, giving us the averaging time that we need to resolve one quantum transitions.
3. The microwave cavity is an understandable, right circular cylinder, which makes it possible to identify the its electromagnetic modes when these are measured in situ

with a method developed for this. This allows us understand, measure and control cavity shifts for the first time.

4. The trap cavity is at 0.1 mK (rather than 4.2 K in the past). This eliminates blackbody photons that would frequency excite the cyclotron ground state.
5. Great signal-to-noise for one-quantum transitions is obtained using electronic feedback to realize the first one-particle self-excited oscillator.

The new methods are powerful enough that we now aspire to a million-times-improved measurement of the antiproton magnetic moment [10] – a moment that is about 500 times smaller than that of the electron.

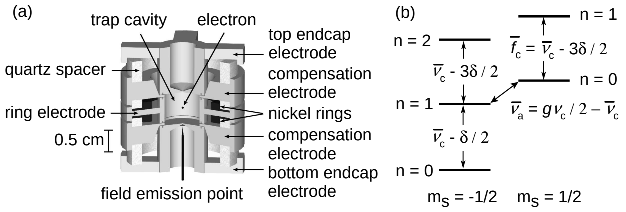

FIGURE 2. Cylindrical Penning trap cavity used to confine a single electron and inhibit spontaneous emission (a), and the cyclotron and spin levels of an electron confined within it (b).

The electron is suspended in a cylindrical Penning trap (Fig. 2a) [11]. The electron starts in one of the two lowest of the cyclotron and spin levels (Fig. 2b), the cyclotron levels being slightly modified (in a well understood and completely measurable way) by the trap potentials and δ comes from special relativity. Feedback is used to produce a one-electron self-excited oscillator [10] which serves as a detector for one quantum cyclotron and spin transitions, with a spin flip (Fig. 3a) and a cyclotron transition (Fig. 3b) clearly resolved [8, 10]. Quantum jump spectroscopy (measuring the quantum jumps per attempt to drive them as a function of drive frequency) gives resonance lineshapes for the cyclotron frequency $\bar{f}_c$ (Fig. 3c) and the anomaly frequency $\bar{\nu}_a$ (Fig. 3d). The line shape comes from thermal axial motion within a magnetic bottle gradient [15].

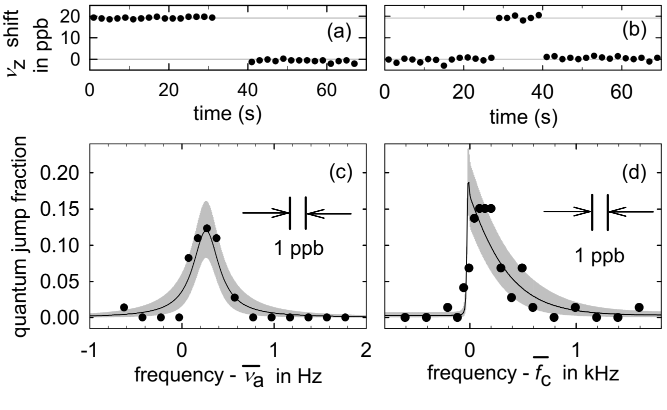

FIGURE 3. Sample $\bar{\nu}_z$ shifts for a spin flip (a) and for a one-quantum cyclotron excitation (b). Quantum jump spectroscopy lineshapes for anomaly (c) and cyclotron (d) transitions, with a maximum likelihood fit to the calculated lineshapes (solid). The bands indicate 68% confidence limits for distributions of measurements about the fit values.

The electron g is deduced from the anomaly and spin-up cyclotron frequencies ($\bar{\nu}_a \approx$ 173 MHz and $\bar{f}_c$ in Fig. 2b) using

$$\frac{g}{2} = \frac{\bar{\nu}_c + \bar{\nu}_a}{\nu_c} \simeq 1 + \frac{\bar{\nu}_a - \bar{\nu}_z^2/(2\bar{f}_c)}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)}, \tag{2}$$

where $\bar{\nu}_z$ is the measured frequency of the electron's harmonic oscillation along the magnetic field direction. The new value for the electron magnetic moment is given by

$$g/2 = 1.001\,159\,652\,180\,85\,(76) \quad (0.76 \text{ ppt}). \tag{3}$$

The standard deviation, about six times smaller than from any previous measurement, arises mostly from an imperfect lineshape model and cavity shifts.

4. DETERMINATION OF THE FINE STRUCTURE CONSTANT

The new electron g, together with QED theory, determine the fine structure constant, α, about ten times more accurately than does any rival method. QED provides an asymptotic series relating g and α,

$$\begin{aligned}\frac{g}{2} = 1 \quad &+ \quad C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 \\ &+ \quad \ldots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},\end{aligned} \tag{4}$$

with hadronic and weak contributions added, and assuming no electron substructure. Impressive calculations, summarized in [2], give exact C_2, C_4 and C_6, a numerical value and uncertainty for C_8, and a small $a_{\mu\tau}$.

Our new value for α [2] comes from the measured g and Eq. 4,

$$\begin{aligned}\alpha^{-1} \quad &= \quad 137.035\,999\,710\,(90)\,(33) \;\; [0.66 \text{ ppb}]\,[0.24 \text{ ppb}], \\ &= \quad 137.035\,999\,710\,(96) \qquad [0.70 \text{ ppb}],\end{aligned} \tag{5}$$

The first line gives the experimental uncertainty first and the QED uncertainty second, including an estimated contribution from a yet uncalculated C_{10} [2]. The total 0.70 ppb uncertainty is ten times smaller than for the next most precise methods (Fig. 1b) – determining α from measured mass ratios, optical frequencies, together with either Rb [12] or Cs [13] recoil velocities.

5. TESTING QED, AND PROBING ELECTRON STRUCTURE

The most stringent test of QED (one of the most demanding comparisons of any calculation and experiment) comes from comparing the measured and calculated $g/2$. The new g, together with α(Cs) or α(Rb) in Eq. 4, give a difference $|\delta g/2| < 15 \times 10^{-12}$. The small uncertainties in $g/2$ will allow a ten times more stringent test if ever the large uncertainties in the independent α values can be reduced. The prototype of modern physics theories is thus tested far more stringently than its inventors could ever have envisioned.

The same comparison of theory and experiment probes the internal structure of the electron [3] – limiting the electron to constituents with a mass $m^* \geq m/\delta g/2 = 34,000$ TeV/c^2, with an electron radius $R < 6 \times 10^{-24}$ m. If this test was limited only by our experimental uncertainty in g, then we could set a limit $m^* > 600$ GeV. These high energy limits seem somewhat remarkable for an experiment carried out at 100 mK.

6. CONCLUSION

In conclusion, greatly improved measurements of the electron magnetic moment and the fine structure constant, and a sensitive probe for internal electron structure, come from resolving the lowest cyclotron and spin levels of a one-electron quantum cyclotron. A self-excited oscillation of the electron reveals one-quantum transitions. A cylindrical Penning trap cavity narrows resonance lines by inhibiting spontaneous emission. Electromagnetic modes of this understandable cavity geometry, probed with synchronized electrons, shift g in a measurable way. The new $g/2$ is shifted from a long accepted value by 1.7 standard deviations, and its fractional accuracy of 7.6×10^{-13} is nearly six times smaller. The new α has an uncertainty ten times smaller than that from any other determination.

The theses of B. D'Urso [16] and B. Odom [17] give many measurement details, and a preliminary analysis is in the latter. S. Peil, D. Enzer, and K. Abdullah contributed to earlier versions of the apparatus. The NSF AMO program provided long-term funding.

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