

The following is an introduction to *Mathematica* for students in a first-year mechanics course. They were used throughout the Fall 2016 semester in Amherst College's *The Newtonian Synthesis* (PHYS-123), taught by Prof. David Hanneke. The introduction consists of a series of tutorials (in the form of *Mathematica* notebooks) and follow-up assignments. These emphasize *Mathematica*'s usefulness for computer algebra, plotting, and numerically integrating equations of motion. As an introduction, many advanced features are left untouched. Problem and figure numbers below refer to our textbook: Young & Freedman's *University Physics*, 13th edition.

1. *Computing*: The laws of physics apply in more cases than we can easily do "by hand" in class or on homework. Throughout the semester, we'll use computational techniques to explore some cases beyond the ones where the algebra is easy. To help in this task, we'll use a software package called *Mathematica*.

*Mathematica* is installed on all College computer except kiosks. Our license allows you to install it on your personal computer should you so desire. Both Windows and Mac are supported. You have to install it off the **Software** drive. Some instructions are available at <https://www.amherst.edu/mm/389493>. You must be on campus to install the software, but you can use it from anywhere if you connect to the license server through the VPN.

There are two programs: *Mathematica* and the *Mathematica Kernel*. The Kernel is a back-end program that automatically runs with the main program. You shouldn't start it on your own.

- Open *Mathematica*, either on a College computer or after installing it on your personal computer.
- Follow along the `Introduction.to.Mathematica.01.nb` notebook.

Use the techniques in the notebook to calculate the following quantities analytically and numerically. Either print out the result or write down the numeric result in scientific notation with three significant figures (for example,  $\pi^{100} = 5.19 \times 10^{49}$ ).

- (a)  $\pi^{(e^3)} - (\pi^e)^3$
- (b) 500! (That's a factorial.)
- (c)  $\cos(3\pi/8) \times \sin(63^\circ)$  (Note that *Mathematica* uses radians by default.)

2. Follow along the `Introduction.to.Mathematica.02.nb` notebook. Use the techniques in the notebook to answer the following.

- (a) For the two vectors  $\vec{A} = 1.2\hat{i} + 2.1\hat{j} + 3.2\hat{k}$  and  $\vec{B} = -4.2\hat{i} + 4.2\hat{j} - 1.7\hat{k}$ , calculate (i) the vector  $\vec{A} - \vec{B}$ , (ii) the magnitude of the vector  $\vec{A} + \vec{B}$ , and (iii) the angle between  $\vec{A}$  and  $\vec{B}$ . (You could do these by hand, but use *Mathematica* and print out your notebook.)
- (b) An object is thrown straight up from  $y_0 = 1.5$  m at  $v_0 = 10$  m/s and falls freely to the ground. Make a `ListPlot` of  $y$  (in meters) versus  $t$  (in seconds) showing its flight at around 20 points.

3. Follow along the `Introduction.to.Mathematica.03.nb` notebook. Use the techniques in the notebook to answer the following.

- (a) Write a simple trig function equivalent to

$$\frac{1 - \cos(2\theta)}{2 \tan^2(\theta - \pi)}$$

- (b) If you multiply out the following expression and group terms with the same power of  $x$ , what is the numeric coefficient of the  $x^2$  term?

$$(3x^2 - 4x + 5)(x^3 - 16x) + x^2 - (x + 2)(x - 5)(x + 8)(x - 4)$$

- (c) Write a function or functions (a genuine *Mathematica* function, as described in the tutorial) that calculates the position as a function of time for the stuntwoman in problem 3.52 (2D projectile motion with  $x_0 = 0$ ,  $y_0 = 30.0$  m,  $v_{x0} = 15.0$  m/s south,  $v_{y0} = 10.0$  m/s). Use it to make a `ListPlot` of  $y$ -versus- $x$  for her motion. Make sure to label your axes properly.

4. Follow along the `Introduction_to_Mathematica_04.nb` notebook. Use the techniques in the notebook to make a plot in part (c) below.
- Imagine a line connecting the center of the Earth to the center of the Moon. Write an expression for the acceleration due to the gravitational attractions to the Earth and Moon for an object along this line. Use the distance from the center of the Earth as the position coordinate  $r$  and be clear about whether a positive acceleration is towards the Earth or the Moon. Your expression should be in terms of this distance  $r$ , the masses of the Earth and Moon, and the distance  $d_{EM}$  between the Earth and Moon. (Ignore any other interaction such as attraction to the Sun or the Earth's rotation.)
  - At what location along this line does the gravitational attraction to the Moon just balance the attraction to the Earth?
  - Plot the acceleration versus distance for positions ranging from the surface of the Earth to the surface of the Moon. The plotted line should end at these surfaces (not inside the Earth or Moon), such that the endpoints of the line show the acceleration due to gravity on their surface (e.g.  $-9.8 \text{ m/s}^2$  for the Earth). To keep the horizontal axis tidy, you might try plotting distance in megameters ( $10^6 \text{ m}$ ) or factor out some other large number. Make sure to label your axes.
5. Follow along the `Introduction_to_Mathematica_05.nb` notebook. Use the numerical integration example of air drag to make a plot showing the time it takes a paper cone to fall two meters as a function of its mass. Show that it approaches the no-drag limit of 0.64 s for sufficiently large mass. (Use the drag force  $f = Dv^2$  with  $D = 0.0025 \text{ kg/m}$ .) Here are a few tips.
- Make a `ListPlot` with a dozen or so points on it.
  - The tutorial notebook contains working air-drag code. You don't have to modify it for this assignment (except the mass of course), though I encourage you to play around a bit.
  - You can access the elements of a list with the `Part` command. The short-hand is the double-brackets `[[ ]]`. For example, to access the first element of list `mylist`, enter `mylist[[1]]`. You can access the last element with `mylist[[-1]]`.
- Use your *Mathematica* skills to reproduce the book's figure 5.27: a comparison of the trajectory of a baseball hit at 50 m/s at an angle of  $35^\circ$  above the horizontal, with and without air drag. Use the drag coefficient  $\frac{1}{2}\rho C_D A = D = 1.3 \times 10^{-3} \text{ kg/m}$ . Use 0.145 kg for the mass of a baseball. Feel free to plot the drag-free version analytically since you can do it exactly. For the version with drag, note that the drag force,  $-Dv^2\hat{v}$ , is always directed opposite the velocity. Since the velocity is changing direction, you will have to simulate the  $x$  and  $y$  dimensions simultaneously and account for the direction of the force. I recommend using the identity  $v^2\hat{v} = v^2\vec{v}/v = v\vec{v}$ . If your current velocity is  $(v_x, v_y)$ , then the drag force is  $(-D\sqrt{v_x^2 + v_y^2}v_x, -D\sqrt{v_x^2 + v_y^2}v_y)$ .
6. Follow along the `Introduction_to_Mathematica_06.nb` notebook. Use the techniques in the notebook to evaluate the following expressions.

(a)  $\int \frac{dx}{\sqrt{a^2+x^2}}$

(b)  $\int_{-\infty}^{\infty} e^{-bx^2} dx \quad (b > 0)$

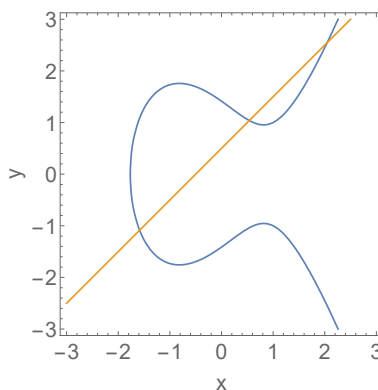
(c)  $\frac{d^2 x^x}{dx^2}$

7. Follow along the `Introduction_to_Mathematica_07.nb` notebook. Use the techniques in the notebook to answer the following.
- For a general 1D elastic collision, object A (mass  $m_A$ , initial velocity  $v_{A1}$ ) collides with object B (mass  $m_B$ , initial velocity  $v_{B1}$ ) and they end up with final velocities  $v_{A2}$  and  $v_{B2}$ . Solve for  $v_{A2}$  and  $v_{B2}$  in terms of  $m_A$ ,  $m_B$ ,  $v_{A1}$ , and  $v_{B1}$ .
  - The curves in the graph below are given by

$$y^2 = x^3 - 2x + 2$$

$$2y = 2x + 1.$$

They intersect at three points. Find the numeric values of the intersection points  $\{x, y\}$  to three digits each. One is  $\{-1.59, -1.09\}$ .



8. Look over `Introduction_to_Mathematica_08.nb`, which introduces vector multiplication in *Mathematica*. Feel free to use it to check results of other problems, though you should make sure you can do cross products “by hand” as well.
9. The *Mathematica* notebook `harmonic_motion.nb` provides a numerical integrator for a 1D damped simple harmonic oscillator. Use it, along with your *Mathematica* skills, to **do one of the following**. (The damped harmonic oscillator is exactly solvable, but sometimes these numeric versions give a quick look at the behavior.)

- (easy) Make a plot of displacement versus time for a damped harmonic oscillator in four cases: very underdamped (e.g. damping equal to 10% of the critical value), slightly underdamped (e.g. damping of half the critical value), critically damped, and overdamped. Put all four on the same axes (similar to figure 14.26, but that’s two underdamped cases).
- (medium) The Romer Art Machine in the department is essentially two independent pendulums that control the  $x$  and  $y$  motion of a pen. Use your knowledge of pendulum oscillations to make an interesting design. Feel free to adjust the positions of the pendulum weights and the counter-weights on the horizontal linking bars. Simulate the behavior of the Art Machine, ideally producing a figure close to your real-life figure. Prof. Romer’s original paper about the machine is R. H. Romer, *A Double Pendulum “Art Machine”*, *American Journal of Physics* **38**, 1116–1121 (1970).
- (hard) Add a driving term and find the steady-state amplitude at several drive frequencies to reproduce points tracing out the resonance curve (amplitude versus drive frequency) for damping equal to 10% of the critical value (the  $b = 0.2\sqrt{km}$  case in figure 14.28).

